CFD Homework

#1

(Due 1384/8/1)

1- Determine the values of x and y to make the following PDE parabolic, elliptic, or hyperbolic.

$$x\frac{\partial^2 u}{\partial x^2}+x\frac{\partial^2 u}{\partial x \partial y}+y\frac{\partial^2 u}{\partial y^2}=0$$

2- Boundery layer equations for 2-D, incompressible flows is presented. Determine the type of this system of equations. (*Re* is Reynolds number)
(Hint: Refer to Computational method for engineers.by:Hoffman, Vol. 1)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2}$$

3-Verify the following expression.

$$\left(\frac{\partial^n u}{\partial x^n}\right)_{i,j}=\frac{\Delta^n u_{i,j}}{(\Delta x)^n}+O(\Delta x)$$

4-Derive fisrt order forward difference for the mixed derivative $\left(\frac{\partial^2 u}{\partial x \partial y}\right)_{i,j}$

5-Calcuatde the first derivative of $f(x)=\tan\left(\frac{\pi x}{4}\right)$ at $x=1.5$, using fisrt order forward difference and first order backward difference approximations. Use three step size 0.01 , 0.5 , 0.8 and discuss about the error of results.

6- Calculate the partial derivative $\left(\frac{\partial u}{\partial y}\right)_{i,j}$ in a non uniform grid. Use Taylor series expantion.