

## Solution of 1D parabolic partial differential equation

### Transient heating of a sphere-implicit method

#### Model problem 2

$$T_t := \mathbf{T}_{xx} + \frac{2}{x}T_x$$

$$\begin{aligned} \text{Bi} &:= 1 & M &:= 10 & N &:= 5 & L &:= 1 & T &:= 1 \\ f(x) &:= 0 \end{aligned}$$

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Thomas(a,d,b,r) :=
  n ← rows(a) - 1
  a_0 ← a_0 / d_0
  r_0 ← r_0 / d_0
  for i ∈ 1..n - 1
    denom ← d_i - b_i · a_{i-1}
    a_i ← a_i / denom
    r_i ← (r_i - b_i · r_{i-1}) / denom
  r_n ← (r_n - b_n · r_{n-1}) / (d_n - b_n · a_{n-1})
  x_n ← r_n
  for i ∈ n - 1..0
    x_i ← r_i - a_i · x_{i+1}
  x

```

Tempimpl(f, Bi, M, N, L, T) :=

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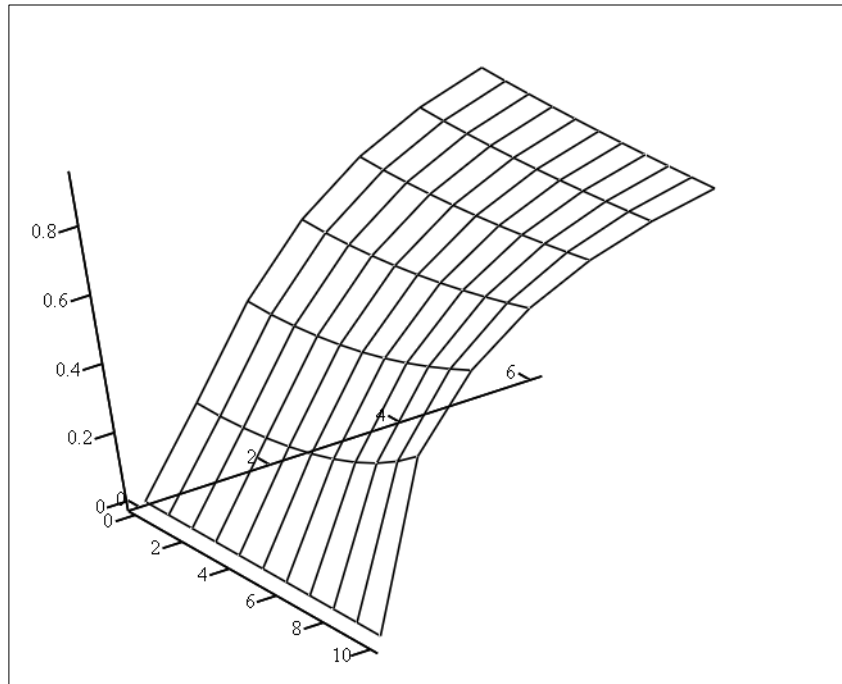
dx ←  $\frac{L}{M}$ 
dt ←  $\frac{T}{N}$ 
a0 ← 1
d0 ← -1
b0 ← 0
r0 ← 0
dM ← -(1 + Bi·dx)
bM ← 1
rM ← -Bi·dx
aM ← 0
for i ∈ 1..M - 1
    xi ← dx·i
    ai ←  $\frac{dt}{dx^2} + \frac{dt}{x_i \cdot dx}$ 
    di ←  $- \left( 1 + 2 \cdot \frac{dt}{dx^2} \right)$ 
    bi ←  $\frac{dt}{dx^2} - \frac{dt}{x_i \cdot dx}$ 
    ri ← -f(xi)
for n ∈ 0..N
    Temp ← Thomas(a, d, b, r)
    for i ∈ 1..M - 1
        ri ← -Tempi
        resi,n+1 ← Tempi
        xi ← dx·i
        ai ←  $\frac{dt}{dx^2} + \frac{dt}{x_i \cdot dx}$ 
        di ←  $- \left( 1 + 2 \cdot \frac{dt}{dx^2} \right)$ 
        bi ←  $\frac{dt}{dx^2} - \frac{dt}{x_i \cdot dx}$ 

```

$$\begin{array}{|l}
 \int_0^1 dx^2 \quad x_i \cdot dx \\
 \text{res}_{0,n+1} \leftarrow \text{res}_{1,n+1} \\
 \text{res}_{M,n+1} \leftarrow \text{Temp}_M \\
 r_M \leftarrow -Bi \cdot dx \\
 \text{res}
 \end{array}$$

Tempimpl(f, Bi, M, N, L, T) =

	0	1	2	3	4	5	6
0	0	0.225	0.459	0.635	0.756	0.838	0.892
1	0	0.225	0.459	0.635	0.756	0.838	0.892
2	0	0.231	0.465	0.64	0.76	0.84	0.893
3	0	0.241	0.475	0.647	0.764	0.843	0.896
4	0	0.254	0.489	0.657	0.771	0.848	0.899
5	0	0.273	0.506	0.67	0.78	0.854	0.903
6	0	0.296	0.528	0.685	0.791	0.861	0.907
7	0	0.326	0.553	0.703	0.802	0.869	0.913
8	0	0.363	0.582	0.723	0.816	0.877	0.918
9	0	0.407	0.614	0.745	0.83	0.887	0.925
10	0	0.461	0.649	0.768	0.846	0.897	0.932



Tempimpl(f, Bi, M, N, L, T)